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The Effect of Asphalt Layer Thickness Variations on Pavement Evaluation Using the Falling Weight Deflectometer

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This paper presents a model for statistically correcting asphalt layer stiffness moduli determined from the Falling Weight Deflectometer (FWD) test to account for asphalt layer thickness variations. Both random variations in asphalt layer thickness and a difference between the mean asphalt layer thickness and the constant value of asphalt layer thickness used in the back-analysis procedure (ie mean thickness error) are included and the model has been validated using published data. The model shows that the mean value of back-calculated asphaltic material stiffness modulus is primarily influenced by incorrect assumptions regarding the mean asphalt layer thickness, whereas the standard deviation of the back-calculated asphaltic material stiffness modulus is also sensitive to variations in the thickness of the asphalt layer around the mean level.

Keywords: FWD, Stiffness Modulus, Thickness

INTRODUCTION

The Falling Weight Deflectometer (FWD) enables a detailed pavement investigation to be performed using a rapid, non-destructive test method. The FWD test involves applying a transient load to the pavement by dropping a mass from a specified height and measuring the dynamic velocity response of the pavement surface at a number of radial positions from the applied load. Typically, these dynamic velocity responses are integrated and the peaks are recorded to give an equivalent quasi-static surface deflection bowl (Sorenson and Hayven, 1982). The FWD applies a load of similar magnitude to that of a typical heavy vehicle tyre and the duration of the load pulse corresponds to a wheel velocity of 60–80 km/h for the upper pavement layer. However, Sebaaly *et. al.* (1991) noted that although the FWD is the most effective NDT device currently available, it does not fully represent the loading conditions generated by a moving truck load. For example, the frequency content of the FWD signal is from 2–200Hz whereas the frequency content of the load history from a truck travelling at typical highway speed is from 1–20Hz. Nonetheless, the FWD represents the current state-of-the-art in accurate deflection testing (Hoy-

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inck *et. al.*, 1992). Deflection results from the FWD can be used directly to give an indication of pavement condition (Brown, *et. al.*, 1986) but, more usually, the surface deflections are used in a back-analysis procedure to give information on the in-situ stiffness moduli of the various pavement layers.

The subject of back-analysis of pavement stiffness moduli from measured quasi-static surface deflection bowls has recently received much attention in the literature due to the increased popularity of the pavement surface deflection test as a method of pavement structural evaluation. This has resulted in a proliferation of back calculation models (Harrichandran *et. al*, 1994; Maestas and Mamlouk, 1991). These back calculation models can be broadly classified into two groups. The majority involve numerous calls to a linear elastic pavement analysis program (Harrichandran *et. al*, 1994), or a non-linear elastic pavement analysis program (de Almeida *et. al.*, 1994). The second group involve matching measured deflection bowls to a database of deflection bowls previously computed, perhaps through a neural network algorithm (Harrichandran *et. al*, 1994). Typically, most back-analysis procedures use a static pavement response model to reproduce the deflection bowl generated from the dynamic FWD surface deflection test. This simplification was examined by Tam and Brown (1989) who concluded that the inertial effects of the pavement were insignificant and a static model could be used. This conclusion is consistent with research by Hardy (1990), who found that increasing the density of the layers in a dynamic layered elastic analysis had virtually no effect on the horizontal strain response at the base of the asphalt layer.

Ideally, variations in stiffness moduli obtained from back-analysis of FWD surface deflections would be only due to variations in material properties. However, there are a number of additional factors that affect variations in back-calculated layer stiffness moduli including random variations in layer thicknesses. Harrichandran *et. al.* (1994) examined the effects of incorrect asphalt layer thickness specification on back-calculated stiffness moduli using 3 different back-analysis procedures. They presented their results in terms of an error in stiffness modulus of the asphaltic material [100%×(estimated stiffness modulus – actual stiffness modulus) / actual stiffness modulus] as a function of percentage error in asphalt layer thickness assumed in the back-analysis procedure. They found that an incorrect asphalt layer thickness in the range ±40% would cause an error in the back-calculated asphaltic material stiffness modulus of between −50% and +200% compared to the true material stiffness modulus.

Briggs *et. al.* (1992) examined the effects of asphalt layer thickness variations on back-calculated stiffness moduli. They used FWD deflection data from 4 Strategic Highway Research Program (SHRP) sites where the thickness of the asphaltic material varied from approximately 50mm to 230mm. Detailed thickness information for the asphaltic material and granular material was obtained using Ground Penetrating Radar (GPR). Back-analysis was performed using these thickness variations and the results were compared to back-analysis performed using assumed thicknesses from the SHRP data base. They found that variations in layer thicknesses were large enough to cause up to a 100% error in the back-calculated stiffness modulus of the asphaltic material.

This paper examines the effects of asphalt layer thickness variations on back-calculated asphaltic material stiffness moduli and presents a model for adjusting statistical stiffness modulus estimates due to random variations in the thickness of the asphalt layer and an incorrect mean asphalt layer thickness specified in a back-analysis procedure.

CORRECTION FOR VARIATIONS IN ASPHALT LAYER THICKNESS

It will be assumed that variations in asphalt layer thickness only affect the back-calculated stiffness modulus of the asphaltic material. In reality, variations in asphalt layer thickness will also affect the back-calculated stiffness moduli of the lower layer materials which, to a large extent, depend on the details of the procedure used in the back-analysis procedure. Consequently, this simplified approach can be considered to give an upper bound estimate for the corrections to the stiffness modulus of the asphaltic material due to variations in the thickness of the asphalt layer and incorrect assumptions regarding the mean asphalt layer thickness. For ease, the terms used are defined at the end of the paper in the Notation Section.

Thickness and stiffness modulus coupling

To correct for the effects of variations in asphalt layer thickness on the back-calculated asphaltic material stiffness modulus it is necessary to know how the thickness and stiffness modulus couple together to influence surface deflections. The degree of coupling was investigated using a simplified model of a three layer flexible pavement structure. The thickness and stiffness modulus of the surface layer (representing the asphaltic material) was varied between 75mm and 300mm and between 2,000MPa and 20,000MPa respectively (Poisson's ratio was taken to be 0.35). A standard granular sub-base and subgrade were assumed for the second and third layers with stiffness moduli of 100MPa and 50MPa respectively. Poisson's ratio was taken to be 0.4 for both layers and the thickness of the sub-base was taken to be 250mm (it was assumed that the subgrade was a semi-infinite half-space). A standard layered elastic model was used to calculate the pavement surface deflection due to simulated FWD loading of magnitude 49.5kN applied over a circular contact area of diameter 300mm for the various asphalt layer thicknesses and stiffness modulus combinations. It was assumed that the effective bending stiffness of the system (S_b) is proportional to the product of the stiffness modulus (E) of the asphalt layer multiplied by the thickness (h) of the asphaltic layer raised to some exponent (m), ie:

$$S_b \propto Eh^m \tag{1}$$

It should be noted that if the exponent m is taken to be 3 the system reduces to the bending stiffness of an elastic beam (or plate). The difference between the calculated deflection directly under the load (d_1) and the calculated deflection a radial distance 300mm from the load (d_2) is then taken to be inversely proportional to the effective bending stiffness raised to another exponent (p), ie:

$$(d_1 - d_2) \propto \frac{1}{(Eh^m)^p} \tag{2}$$

The deflection difference $(d_1 - d_2)$ has been related to the effective bending stiffness because previous research has shown that surface deflection differences in the vicinity of the applied load relate primarily to the condition of the upper pavement layer which, in this case, is asphaltic material (Brown *et. al.*, 1986).

It can be seen that Equation (2) gives a straight line of gradient $-mp$ when $\log_{10}(d_1 - d_2)$ is plotted against $\log_{10}(h)$ for a particular value of E, and a straight line of gradient $-p$ when $\log_{10}(d_1 - d_2)$ plotted against $\log_{10}(E)$ for a particular value of h. Using a least squares approach is was found that the values of m varied between 1.5 (low stiffness, large thicknesses) and 3.7 (high stiffness, small thicknesses) and the values of p varied between 0.5 (small thicknesses) and 0.9 (large thicknesses).

Figure 1 shows the fitted relationship between deflection difference $d_1 - d_2$, asphalt layer thickness and asphalt material stiffness modulus (solid lines) together with the corresponding data calculated using the layered elastic model. It can be seen from this figure that a reasonable fit has been achieved over quite a large range of asphaltic material stiffness moduli and asphalt layer thicknesses. A simple model of the form:

$$m = C_1 \left(\frac{E}{E_{ref}}\right)^{C_2} \left(\frac{h}{h_{ref}}\right)^{C_3} \tag{3}$$

was fitted (using a least squares technique) to the results to enable m to be predicted for given combinations of asphalt layer thickness (in mm) and stiffness modulus (in GPa). The resulting values of C_1, C_2 and C_3 were found to be 2.29, 0.17 and –0.38, E_{ref} and h_{ref} were taken to be 10GPa and 200mm respectively. Using this approach the accuracy of the predicted values of m (characterised by the percentage difference between the predicted and the fitted values of m – see Figure 1) was found to be within ± 6% which was considered to be satisfactory. This simplified approach has been used in the following analysis.

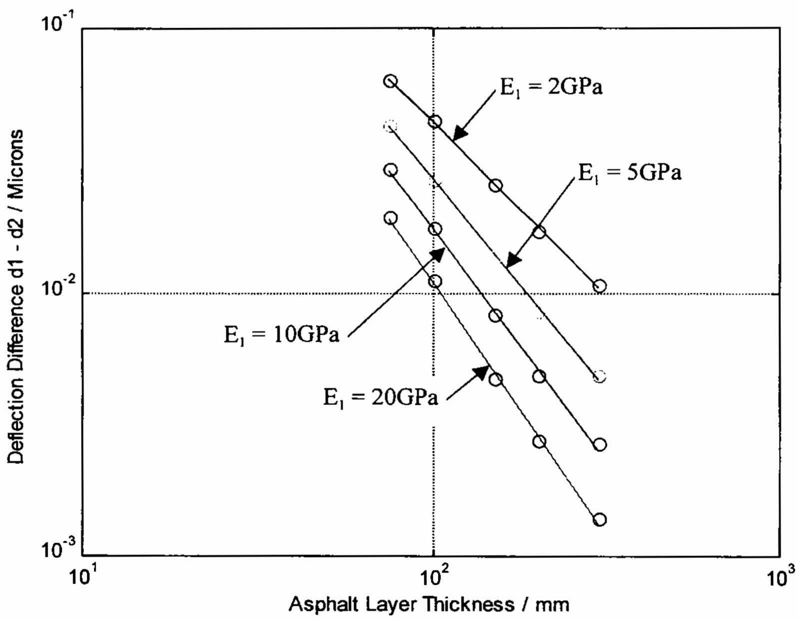

FIGURE 1 Predicted deflection difference ($d_1 - d_2$) versus asphalt layer thickness

Modelling

Assuming that both the asphaltic material stiffness modulus and asphalt layer thickness profiles are uncorrelated random variables, the effective bending stiffness (Equation (1)) may be written as:

$$S_b(x) = K(\overline{E} + \Delta E(x))^p(\overline{h} + h_0 + \Delta h(x))^{mp} \quad (4)$$

where

$\overline{E}$ is the mean asphalt stiffness modulus,

$\Delta E(x)$ is a zero mean variable, having variance $\sigma^2_{\Delta E}$, representing asphaltic material stiffness modulus variations,

$(\overline{h} + h_0)$ is the mean asphalt layer thickness (h_0 can be interpreted as the error in mean asphalt layer thickness),

$\Delta h(x)$ is a zero mean variable, having variance $\sigma^2_{\Delta h}$, representing asphalt layer thickness variations,

x is the longitudinal distance along the pavement, and

K, m and p are constants.

If a constant asphalt layer thickness, $\overline{h}$, is assumed in the FWD analysis and there are no assumed asphalt layer thickness variations $(\sigma^2_{\Delta h} = 0)$ and no assumed errors in the mean asphalt layer thickness ($h_0 = 0$) Equation (4) reduces to:

$$S_b(x) = K(\overline{E} + \Delta E(x))^p \overline{h}^{mp} = K E^P_{FWD}(x) \overline{h}^{mp} \quad (5)$$

Equating Equations (4) and (5) gives the stiffness modulus of the asphaltic material determined from a back-analysis procedure in terms of the variations in asphalt layer thickness and the variations in stiffness modulus of the asphaltic material:

$$\frac{E_{FWD}(x)}{\overline{E}} = \left\{1 + \frac{\Delta E(x)}{\overline{E}}\right\}\left\{1 + \frac{h_0}{\overline{h}} + \frac{\Delta h(x)}{\overline{h}}\right\}^m \quad (6)$$

Using Equation (6) it can be shown that the mean square value $\Phi^2_{E_{FWD}}$ of the FWD back-calculated layer stiffness modulus is given by:

$$\frac{\Phi^2_{E_{FWD}}}{\overline{E}^2} = \mathrm{E}[(E_{FWD})^2] = \mathrm{E}\left[\left\{1 + \frac{\Delta E(x)}{\overline{E}}\right\}^2 \left\{1 + \frac{h_0}{\overline{h}} + \frac{\Delta h(x)}{\overline{h}}\right\}^{2m}\right] \quad (7)$$

where E[ ] is the expectation operator.

Expanding Equation (7) using a binomial approximation (assuming m to be constant) and setting the cross-product terms (eg $\mathrm{E}[\Delta E \Delta h]$) to zero gives the mean square FWD stiffness modulus variation in terms of the true material stiffness modulus variations and asphalt layer thickness variations:

$$\frac{\Phi^2_{E_{FWD}}}{\overline{E}^2} = \left\{1 + \left(\frac{\sigma_{\Delta E}}{\overline{E}}\right)^2\right\}\left\{1 + 2m\left(\frac{h_0}{\overline{h}}\right) + m(2m-1)\left[\left(\frac{h_0}{\overline{h}}\right)^2 + \left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2\right] + \cdots\right\} \quad (8)$$

It can be seen from Equation (8) that the mean square of the FWD back-calculated asphaltic material stiffness modulus depends on the material variability $(\sigma_{\Delta E}/\overline{E})$, thickness variability $(\sigma_{\Delta h}/\overline{h})$ and a measure of the error in mean asphalt layer thickness $(h_0/\overline{h})$.

A similar expression can be obtained for the mean FWD stiffness modulus in terms of the true material stiffness modulus variations and asphalt layer thickness variations by calculating the expected value of Equation (6) giving:

$$\frac{\mu_{E_{FWD}}}{\overline{E}} = 1 + m\left(\frac{h_0}{\overline{h}}\right) + \frac{m(m-1)}{2}\left\{\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2 + \left(\frac{h_0}{\overline{h}}\right)^2\right\} + \cdots \quad (9)$$

Combining Equations (8) and (9) and setting higher order terms to zero, the following expression can be obtained for the standard deviation of the FWD stiffness modulus variation in terms of the true material stiffness modulus standard deviation and asphalt layer thickness variations:

$$\frac{\sigma_{E_{FWD}}}{\sigma_{\Delta E}} \approx \left\{1 + m^2\left(\frac{\overline{E}}{\sigma_{\Delta E}}\right)^2\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2 + 2m\left(\frac{h_0}{\overline{h}}\right)\left[1 + m(m-1)\left(\frac{\overline{E}}{\sigma_{\Delta E}}\right)^2\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2\right] + m(2m-1)\left(\frac{h_0}{\overline{h}}\right)^2\left[1 + \frac{m(m-1)(2m-3)}{(2m-1)}\left(\frac{\overline{E}}{\sigma_{\Delta E}}\right)^2\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2\right] + m(2m-1)\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2\left[1 + \frac{m(m-1)(m-2)}{3(2m-1)}\left(\frac{\overline{E}}{\sigma_{\Delta E}}\right)^2\left(\frac{\sigma_{\Delta h}}{\overline{h}}\right)^2\right]\right\}^{\frac{1}{2}} \quad (10)$$

The accuracy of Equations (9) and (10) was assessed using the following procedure. Firstly, for a particular value of $(h_0/\overline{h})$, $(\sigma_{\Delta E}/\overline{E})$ and $(\sigma_{\Delta h}/\overline{h})$, Equation (6) was used to simulate a stiffness modulus history obtained from FWD testing at 1,000 positions along the pavement (E_{FWD}). To achieve this Δ_E and Δ_h in Equation (6) were treated as uncorrelated random zero mean variables with Gaussian (normal) probability distributions (and standard deviations $\sigma_{\Delta E}$ and $\sigma_{\Delta h}$) and m was taken to be 2.3. From the resulting history of 1,000 points, the mean and standard deviation were calculated. This procedure was repeated a number of times using different values of $(h_0/\overline{h})$, $(\sigma_{\Delta E}/\overline{E})$ and $(\sigma_{\Delta h}/\overline{h})$ in the ranges $-0.15 \leq (h_0/\overline{h}) \leq 0.15$, $0 \leq (\sigma_{\Delta E}/\overline{E}) \leq 0.15$ and $0 \leq (\sigma_{\Delta h}/\overline{h}) \leq 0.15$. In order to compare the estimates given by Equations (9) and (10) with the values calculated using the above procedure a mean error and a standard deviation error were used, defined as:

$$\textit{Mean error} = \left(\frac{\mu_{E_{FWD}}}{\mathrm{E}[E_{FWD}]} - 1\right) \times 100\% \quad (11)$$

Standard deviation error =

$$\left(\frac{\sigma_{E_{FWD}}}{\sqrt{\mathrm{E}[E_{FWD}^2] - (\mathrm{E}[E_{FWD}])^2}} - 1\right) \times 100\% \tag{12}$$

where

$\mu_{E_{FWD}}$ is calculated using the first 4 terms in Equation (9),
$\mathrm{E}[E_{FWD}]$ is calculated from Equation (6),
$\sigma_{E_{FWD}}$ is calculated using Equation (10), and
$\mathrm{E}[E_{FWD}^2]$ and $\mathrm{E}[E_{FWD}]$ are calculated from Equation (6).

Figure 2 shows a probability histogram for the mean error. It can be seen from this figure that the average mean stiffness modulus error is 0.014% with a standard deviation of 0.087%. From the data shown in Figure 2, there is a 95% probability that the mean FWD stiffness modulus calculated using Equation (9) will lie within –0.14% and +0.23% of the mean stiffness modulus calculated from Equation (6).

The corresponding probability histogram for the standard deviation error is shown in Figure 3. It can be seen from this figure that the average stiffness modulus standard deviation error is –0.27% with a standard deviation of 1.3%. From the data shown in Figure 3, there is a 95% probability that the stiffness modulus standard deviation calculated using Equation (10) will lie within –2.83% and +2.57% of the stiffness modulus standard deviation calculated from Equation (6).

Figure 4 shows $[(\mu_{E_{FWD}}/\overline{E}) - 1] \times 100\%$ plotted against $\sigma_{\Delta h}/\overline{h}$ and $h_0/\overline{h}$ taking $m = 2.3$ where $\mu_{E_{FWD}}/\overline{E}$ has been calculated using Equation (9). The vertical axis in this figure can be interpreted as the percentage error in the mean stiffness modulus of the asphaltic material calculated from FWD results compared to the true mean stiffness modulus of the asphaltic material and will be referred to as a FWD stiffness modulus mean error. It can be seen from this figure that the FWD stiffness modulus mean error is most sensitive to incorrect assumptions regarding the mean thickness of the asphalt layer $(h_0/\overline{h})$ and is hardly affected by asphalt layer thickness variability about a mean level $(\sigma_{\Delta h}/\overline{h})$. It can also be seen from this figure that over the range $-0.15 \leq (h_0/\overline{h}) \leq 0.15$ the FWD stiffness modulus mean error varies between approximately –30% and +40%.

Figure 5 shows $[(\sigma_{E_{FWD}}/\sigma_{\Delta E})-1]\times 100\%$ also plotted against $\sigma_{\Delta h}/\overline{h}$ and $h_0/\overline{h}$ taking $m = 2.3$ and $\sigma_{\Delta E}/\overline{E} = 0.1$ where $\sigma_{E_{FWD}}/\sigma_{\Delta E}$ has been calculated using Equation (10). The vertical axis in this figure can be interpreted as the percentage error in the standard deviation of the stiffness modulus of the asphaltic material calculated from FWD results compared to the true standard deviation of the stiffness modulus of the asphaltic material and will be referred to as a FWD stiffness modulus standard deviation error. It can be seen from this figure that the FWD stiffness modulus standard deviation error is most sensitive to asphalt layer thickness variability about a mean level $(\sigma_{\Delta h}/\overline{h})$ compared to assumptions regarding the mean thickness of the asphalt layer $(h_0/\overline{h})$. It can also be seen from this figure that over the range of $(\sigma_{\Delta h}/\overline{h})$ and $(h_0/\overline{h})$ investigated the FWD stiffness modulus standard deviation error varies between approximately –30% and +340%.

Figure 6 shows the FWD stiffness modulus standard deviation error plotted against $\sigma_{\Delta E}/\overline{E}$ for different ratios of $\sigma_{\Delta h}/\overline{h}$ taking $m = 2.3$ and $h_0 = 0$ (ie no mean asphalt layer thickness error). It can be seen from this figure that, as expected, the error in standard deviation increases for higher levels of asphalt layer thickness variability $(\sigma_{\Delta h}/\overline{h})$ and lower levels of asphalt stiffness modulus variability $(\sigma_{\Delta E}/\overline{E})$. For example, assuming that $\sigma_{\Delta E}/\overline{E} = 0.1$ (ie the standard deviation is 10% of the mean) it can be seen from Figure 6 that the FWD stiffness modulus standard deviation error is 150% assuming that $\sigma_{\Delta h}/\overline{h} = 0.1$.

Validation

Harrichandran *et. al.* (1994) examined the effects of incorrect asphalt layer thickness specification on back-calculated stiffness moduli using 3 different back-analysis procedures that do not automatically correct for incorrect layer thicknesses. Their results are shown in Figure 7 in terms of a percentage error in stiffness modulus of the asphaltic material

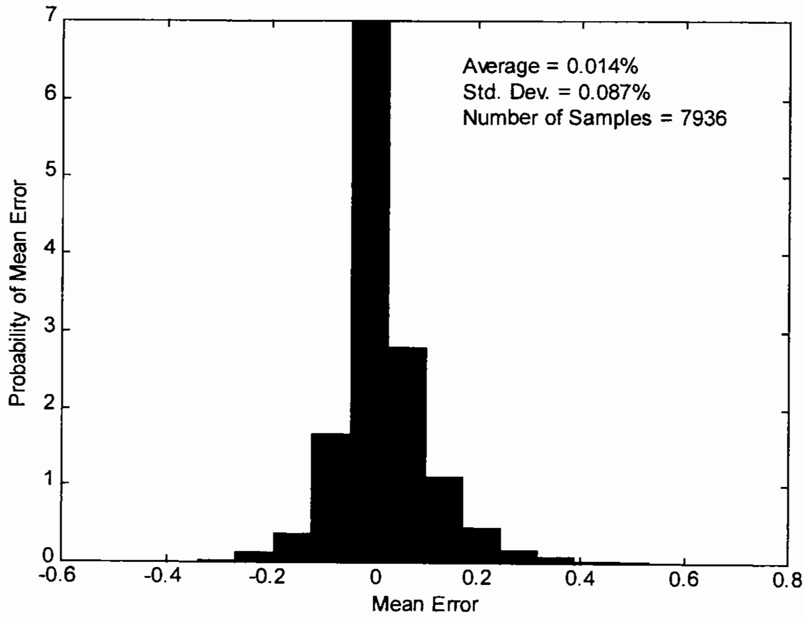

FIGURE 2 Mean stiffness modulus error probability distribution

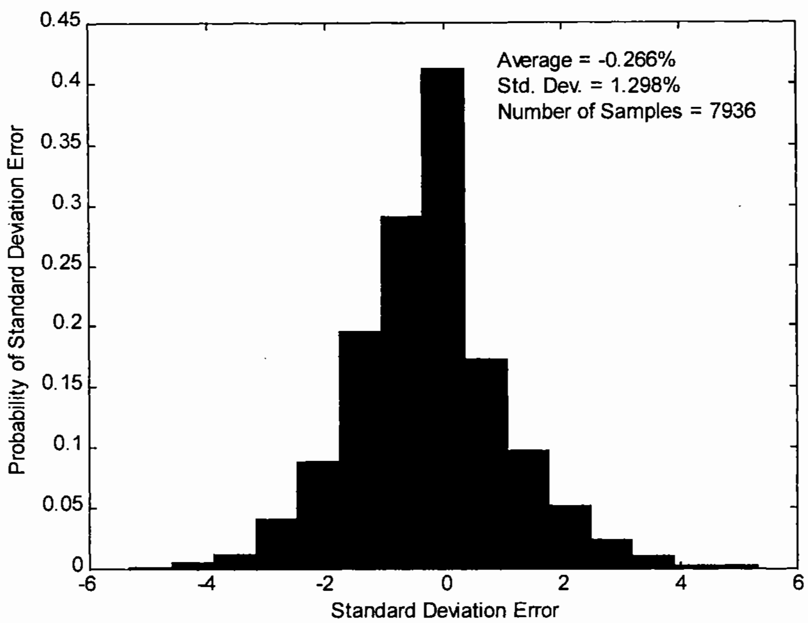

FIGURE 3 Standard deviation stiffness modulus error probability distribution

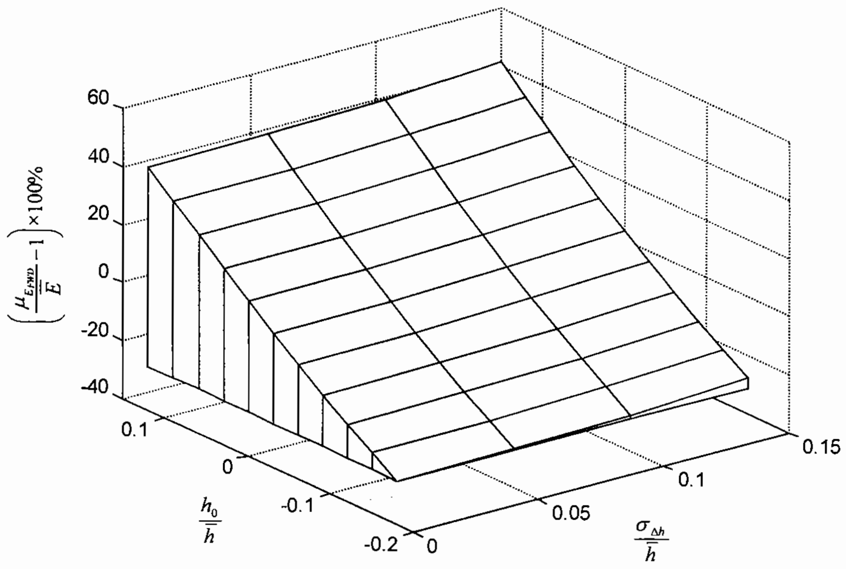

FIGURE 4 Predicted effect of asphalt layer thickness variations on mean back-calculated asphaltic material stiffness modulus as a function of asphalt layer thickness variability

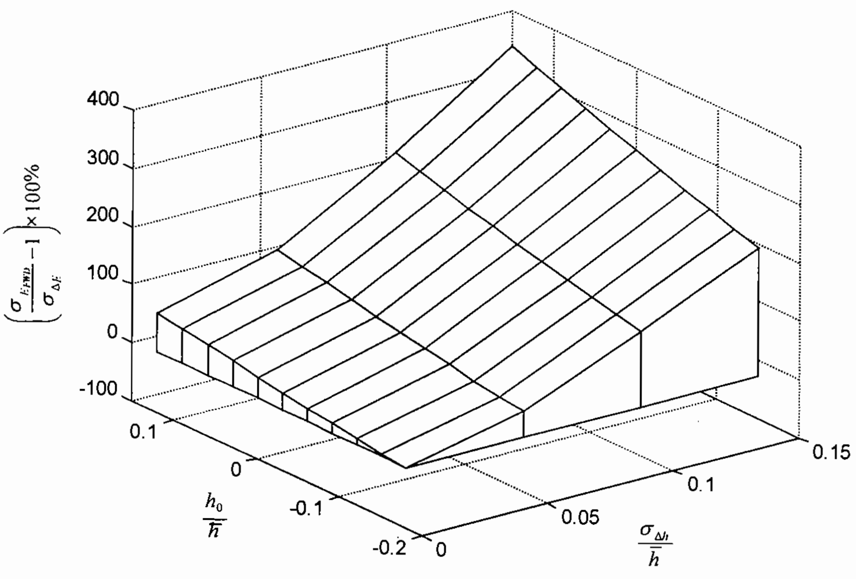

FIGURE 5 Predicted effect of asphalt layer thickness variations on back-calculated asphaltic material stiffness modulus standard deviation as a function of asphalt layer thickness variability

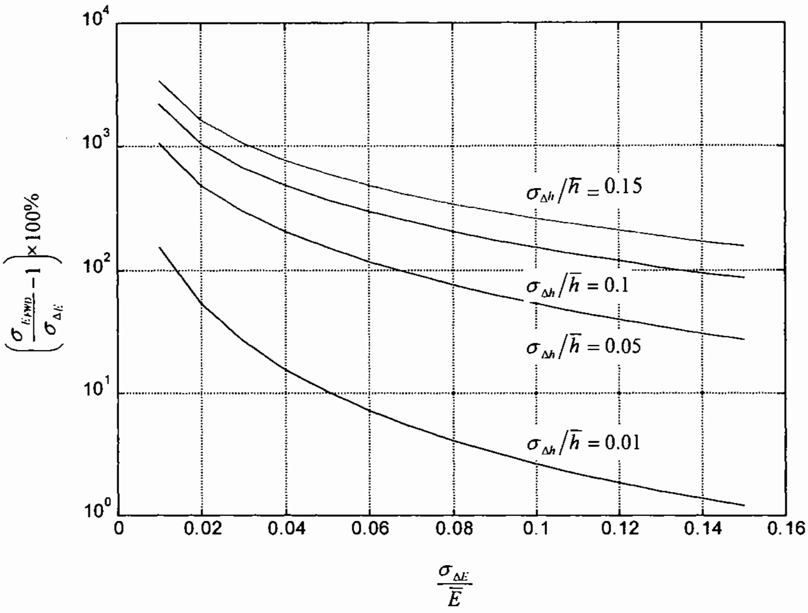

FIGURE 6 Predicted effect of asphalt layer thickness variations on back-calculated asphaltic material stiffness modulus standard deviation as a function of asphalt stiffness modulus variability

[100%×(estimated stiffness modulus – actual stiffness modulus) / actual stiffness modulus] plotted as a function of percentage error in asphalt layer thickness assumed in the back-analysis procedure. The deflection bowls were generated using the CHEVRON computer program with the following parameters; 127mm asphalt layer thickness, 3.4GPa asphalt layer stiffness modulus, 203mm granular base thickness, 310MPa granular base stiffness modulus and 52MPa subgrade stiffness modulus. Poisson's ratios were taken to be 0.35, 0.4 and 0.45 for the asphaltic, granular and subgrade material respectively and a 40kN load was applied to a circular area of radius 150mm. Also shown in Figure 7 is the predicted error for this structure calculated using the first 4 terms in Equation (9). It can be seen from this figure that agreement is generally good although the prediction tends to slightly overestimate the magnitude of percentage error in stiffness modulus of the asphaltic material for negative asphalt layer thickness errors and slightly underestimate the magnitude of percentage error in stiffness modulus of the asphaltic material for positive asphalt layer thickness errors. This is expected since Equation (9) assumes that all the error in asphalt layer thickness is accounted for in the back-calculated stiffness modulus of the asphaltic material. This is not always the case and Harrichandran *et. al.* (1994) found that, for this pavement structure, there was a similar magnitude error in the stiffness modulus of the granular material. It should also be noted that the parameters used in CHEVRON for the granular layer thickness and stiffness modulus, and Poisson's ratio for the subgrade are slightly different compared to the corresponding parameters used for calculating m in Equation (9). This may also account for some of the differences between the predicted errors and the errors calculated by Harrichandran *et. al.* (1994).

Briggs *et. al.* (1992) examined the effects of asphalt layer thickness variations on back-calculated stiffness moduli. They used FWD deflection data from 4 Strategic Highway Research Program (SHRP) sites where the thickness of the asphaltic material varied from approximately 50mm to 230mm. The sites were nominally of uniform construction and were approximately 150m in length with FWD testing at approximately 15m intervals. A total of 16 FWD tests were performed at each test point (4 different load levels). Detailed thickness information for the asphaltic material and granular material was obtained using Ground Penetrating Radar (GPR). Back-analysis was performed using these thickness variations and the results were compared to back-analysis performed using assumed thicknesses from the SHRP data base.

Results from the 4 sections are shown in Table I in terms of the ratio of mean back-calculated moduli obtained using an assumed thickness $(\mu_{E_{FWD}})$ to mean back-calculated moduli obtained using GPR measured thicknesses $(\overline{E})$ and the ratio of the standard deviation of back-calculated moduli obtained using assumed thickness $(\sigma_{E_{FWD}})$ to the standard deviation of the back-calculated moduli obtained using GPR measured thicknesses $(\sigma_{\Delta E})$. Also shown in this table are the predicted effects calculated using Equations (9) and (10). However, because only 11 data points were used in the determination of $(\overline{E})$ and $(\sigma_{\Delta E})$ the degree of confidence needs to be assessed.

Collop *et. al.* (1999) determined the confidence limits for back-calculated asphaltic material stiffness modulus from FWD tests using a variable number of measurement points using a normalised mean error and a standard deviation error calculated using the following Equations:

$$-\frac{\sigma_{\Delta E} t_{n;\alpha/2}}{(\overline{E})^{true}\sqrt{N}} \leq \frac{(\overline{E})^{true} - \overline{E}}{(\overline{E})^{true}} < \frac{\sigma_{\Delta E} t_{n;\alpha/2}}{(\overline{E})^{true}\sqrt{N}} \qquad (13)$$

$$1 - \sqrt{\frac{\chi^2_{n;\alpha/2}}{n}} \leq \frac{(\sigma_{\Delta E})^{true} - \sigma_{\Delta E}}{(\sigma_{\Delta E})^{true}} < 1 - \sqrt{\frac{\chi^2_{n;1-\alpha/2}}{n}} \qquad (14)$$

where

$(\overline{E})^{true}, (\sigma_{\Delta E})^{true}$ are the values of the mean stiffness modulus and the stiffness modulus standard deviation calculated using an infinite number of data points,

N is the number of data points,

$n = N - 1$,

$\chi^2_{n;\alpha}$ is the 100α percentage point of the χ^2 distribution with n degrees of freedom (Bendat and Piersol, 1971),

$t_{n;\alpha}$ is the 100α percentage point of the student t distribution with n degrees of freedom (Bendat and Piersol, 1971).

Equations (13) and (14) allow the 95[th] percentile confidence limits to be added to the data from Briggs *et. al.* (1992) presented in Table I(shown in brackets).

It should be noted that Equations (13) and (14) assume that the asphalt layer stiffness modulus is a random variable and, consequently, the confidence limits only depend on the number of data points and not the distance between two adjacent points. Consequently, Equations (13) and (14) do not account for any spatial correlation between the stiffness modulus and the pavement tested. If this correlation did exist the confidence limits would depend on the position of the samples within the population as well as the number of samples and an alternative procedure would have to be used.

TABLE I Comparison of data from Briggs *et. al.* (1992) with predictions using Equations (9), (10), (13) and (14)

Section No.	$(\mu_{E_{FWD}}/\overline{E})$ *Briggs et al. (1992)*	$(\mu_{E_{FWD}}/\overline{E})$ *Equation (9)*	$(\sigma_{E_{FWD}}/\sigma_{\Delta E})$ *Briggs et. al. (1992)*	$(\sigma_{E_{FWD}}/\sigma_{\Delta E})$ *Equation (10)*
481109	0.85 (0.80 – 0.89)	0.83	0.76 (0.53 – 1.34)	1.25
481050	0.74 (0.52 – 0.96)	1.04	0.66 (0.46 – 1.16)	1.34
481178	0.99 (0.83 – 1.14)	0.90	0.87 (0.61 – 1.53)	0.92
483559	1.29 (1.22 – 1.35)	1.39	4.97 (3.47 – 8.71)	4.10

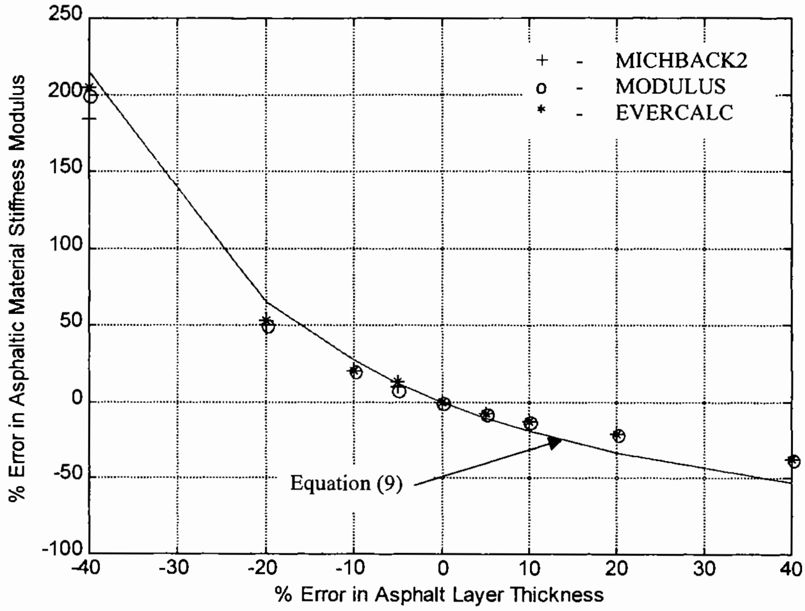

FIGURE 7 Comparison of errors in back-calculated asphaltic material stiffness moduli predicted using Equations (9), (10), (13) and (14) and data from Harrichandran *et. al.* (1994)

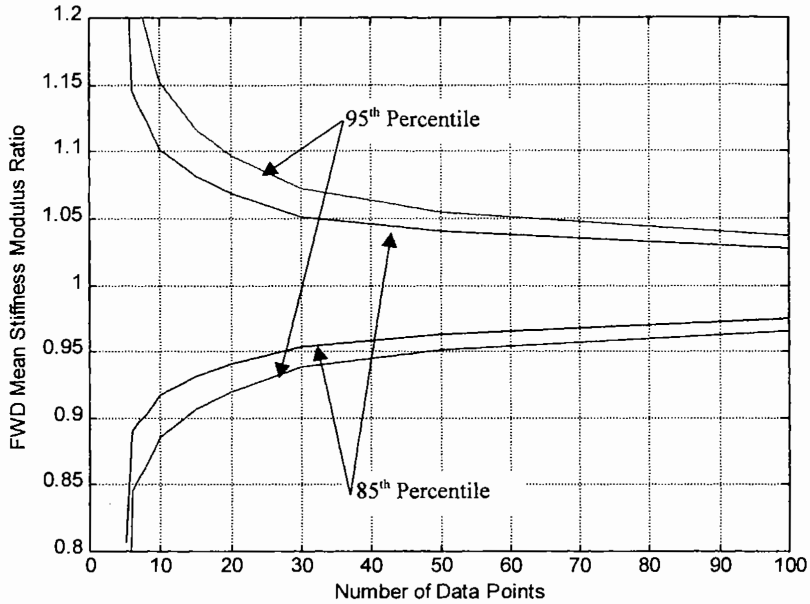

FIGURE 8 Ratio of the mean value of the back-calculated asphaltic material stiffness modulus calculated using a limited number of data points to characterise the asphalt layer thickness to the mean value of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness $(\sigma_{\Delta h}/\overline{h} = 0.07, m = 2.3)$

It can be seen that in 3 cases (sections 481109, 481178 and 483559) the agreement is reasonable between the predicted and measured mean modulus ratios $(\mu_{E_{FWD}}/\overline{E})$ whereas the other case (section 481050) shows poorer agreement. This is likely to be because the average thickness of the asphaltic material for section 481050 was only 58.4mm and the FWD becomes relatively insensitive to variations in asphalt stiffness modulus when the layer is that thin (Briggs *et. al.*, 1992). It is also likely that, for thinner asphaltic layers, errors in the assumed thickness of the lower pavement layers (eg granular sub-base) become relatively more important compared to errors in the assumed thickness of the asphalt layer. The effects of errors in lower layer thickness are not included in the current model.

It can also be seen from Table I that in 2 cases (sections 481178 and 483559) the agreement is reasonably good between the predicted and measured standard deviation ratios $(\sigma_{E_{FWD}}/\sigma_{\Delta E})$. Again section 481050 shows poor agreement (see above) and section 481109 also shows poorer agreement although it should be noted that the predicted value falls within the 95th percentile confidence limits.

Irwin *et. al.* (1989) examined the effect of random variability in layer thicknesses on back-calculated layer stiffness moduli for a 4-layer flexible pavement structure comprising a 76mm surface course (2.1GPa), a 152mm base (310MPa), a 305mm sub-base (145MPa) and a semi-infinite subgrade (52MPa). A single artificial deflection bowl was created, and thirty back-analyses were performed using MODCOMP 2 assigning random thickness errors to each of the three layers above the subgrade. The back-calculated stiffness moduli were then compared to those used to generate the surface deflection bowl. Results showed that the variation in asphalt layer stiffness modulus, characterised in terms of a Coefficient of Variation (COV) was approximately 12%. To compare their results with Equation (10) a value of m is required. However, it can be observed that the pavement structure used by Irwin *et. al.* (1989) is significantly different from the structure used in this paper which is likely to significantly influence the value of m. Therefore, a direct comparison is not possible although it should be noted that if a value of $m = 1.5$ is used (this is typical for asphaltic material with a low stiffness modulus) Equation (10) predicts a COV of approximately 12%.

EFFECT OF A LIMITED NUMBER OF ASPHALT LAYER THICKNESS MEASUREMENTS

It has been shown above that Equations (9) and (10) can be used to determine the effect of incorrect mean asphalt layer thickness assumptions and variations in asphalt layer thickness on back-calculated asphalt layer stiffness moduli assuming that the mean and standard deviation of the asphalt layer thickness have been determined accurately (ie using a large number of data points). Although this is possible using techniques such as Ground Penetrating Radar (GPR) it is more likely that a limited number of measurements (obtained from coring etc) will be used to determine the mean and standard deviation of the asphalt layer thickness. This section investigates the effects of using a limited number of asphalt layer thickness measurements on the back-calculated asphalt layer moduli.

Equations (13) and (14) can also be used to calculate the confidence interval associated with a limited number of asphalt layer thickness measurements by simply replacing the asphalt stiffness modulus with asphalt layer thickness. For example, assuming a typical level of asphalt layer thickness variability (Collop and Cebon, 1995), $\sigma_{\Delta h}/\overline{h} = 0.07$, Equation (13) can be used to predict that if 10 thickness samples are taken, the difference between the estimated mean asphalt layer thickness and the true mean asphalt layer thickness is between approximately ±5.2% for a confidence level of 95% and between approximately ±3.7% for a confidence level of 85%. Similarly Equation (14) predicts that if 10 thickness samples are taken, the difference between the estimated asphalt layer thickness standard deviation and the true asphalt layer thickness standard deviation is between approximately –47% to +46% for a confidence level of 95% and between approximately –33% to +26% for a confidence level of 85%. Taking the estimated asphalt layer thickness to be $\overline{h}$ and the true mean asphalt layer thickness to be $h_0 + \overline{h}$, the corresponding values of $h_0/\overline{h}$ are –0.05 to +0.055 and –0.036 to +0.038 for confidence levels of 95% and 85% respectively.

Consequently, using Equation (9) together with the extreme values of $h_0/\overline{h}$ and a typical value of $m = 2.3$, the ratio of the mean value of the back-calculated asphaltic material stiffness modulus calculated using 10 data points to characterise the asphalt layer thickness to the mean value of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness is between approximately 0.89 and 1.15 for a confidence level of 95% and between approximately 0.92 and 1.1 for a confidence level of 85% (Figure 8). Assuming a value of $\sigma_{\Delta E}/\overline{E} = 0.1$, Equation (10) predicts that the ratio of the standard deviation of the back-calculated asphaltic material stiffness modulus calculated using 10 data points to characterise the asphalt layer thickness to the standard deviation of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness is between approximately 0.7 and 1.89 for a confidence level of 95% and between approximately –0.76 and 1.57 for a confidence level of 85%.

Figure 8 shows the ratio of the mean value of the back-calculated asphaltic material stiffness modulus calculated using several different limited numbers of data points to characterise the asphalt layer thickness to the mean value of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness for confidence levels of 85% and 95% $(\sigma_{\Delta h}/\overline{h} = 0.07, m = 2.3, \sigma_{\Delta E}/\overline{E} = 0.1)$. It can be seen from this figure that the distance between the confidence limits increases as the number of data points used to characterise the mean asphalt layer thickness and standard deviation of the asphalt layer thickness is reduced. For example it can be seen that if 100 data points are used the ratio is between approximately 0.97 and 1.03 for a confidence interval of 85% compared to approximately 0.92 and 1.1 for the same confidence interval and 10 data points.

Figure 9 shows the ratio of the standard deviation of the back-calculated asphaltic material stiffness modulus calculated using several different limited numbers of data points to characterise the asphalt layer thickness to the standard deviation of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness $(\sigma_{\Delta h}/\overline{h} = 0.07, m = 2.3, \sigma_{\Delta E}/\overline{E} = 0.1)$. It can be seen that the curves in this figure follow a similar general trend to those shown in Figure 8 although that the distance between confidence limits is larger for a given number of data points. For example, it can be seen that if 100 data points are used the ratio is between approximately 0.91 and 1.12 for a confidence interval of 85% compared to approximately 0.76 and 1.57 for the same confidence interval and 10 data points.

It can therefore be seen that, although a relatively large number of FWD tests may be performed in order to reduce the errors in mean asphalt stiffness modulus as a consequence of errors in the FWD and point-to-point variation in asphalt stiffness modulus, nevertheless the accuracy of the mean value and the standard deviation of the back-calculated stiffness modulus of the asphaltic material is also limited by the number of samples used to quantify the mean and standard deviation of the asphalt layer thickness.

DISCUSSION

A model has been presented for statistically correcting back-calculated asphaltic material stiffness moduli for asphalt layer thickness variations. Both random variations in asphalt layer thickness and a difference between the mean asphalt layer thickness and the constant value of asphalt layer thickness used in the back-analysis procedure (ie mean thickness error) are included. Results show that the mean value of back-calculated asphaltic material stiffness modulus is primarily influenced by incorrect assumptions regarding the mean asphalt layer thickness, whereas the standard deviation of the back-calculated asphaltic material stiffness modulus is also sensitive to variations in the thickness of the asphalt layer

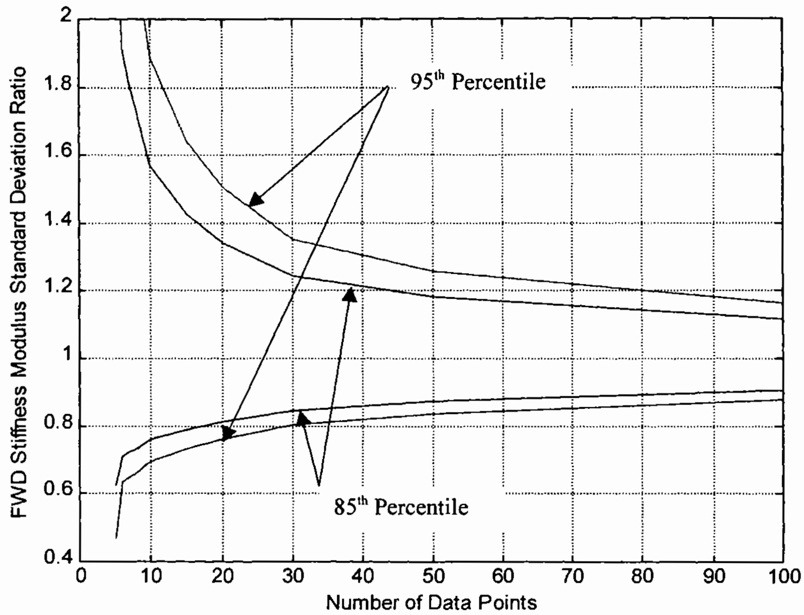

FIGURE 9 Ratio of the standard deviation of the back-calculated asphaltic material stiffness modulus calculated using a limited number of data points to characterise the asphalt layer thickness to the standard deviation of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness $(\sigma_{\Delta h}/\overline{h} = 0.07,\ m = 2.3,\ \sigma_{\Delta E}/\overline{E} = 0.1)$.

around the mean level. Consequently, if an end product testing regime is aimed at characterisation of the mean value of the asphaltic material stiffness modulus the mean asphalt layer thickness should be determined relatively accurately over the length of pavement that is being tested.

In addition to the errors caused by variations in asphalt layer thickness there are also a number of other error sources that can be divided into 2 groups; systematic errors and random errors. Systematic errors will introduce a bias into the process and can be removed by eliminating the source of the bias. Sources of systematic errors include: inapplicability of assumptions in the back-analysis procedure, deviation of the contact pressure from a uniform distribution and a temporal variation in material properties caused by thermal and/or suction gradients (Siddharthan *et. al.*, 1991). Random errors are the result of random variations in the measurements of pavement materials and include; errors in the surface deflection measurements and errors in the measurement of the applied load. However, unlike systematic errors the effects of random errors can be minimised by repeated testing at the same point and averaging the results.

Irwin *et. al.* (1989) examined the effects of measurement accuracy on back-calculated stiffness moduli for a thin flexible pavement (75mm asphalt). Thirty deflection bowls (each containing six surface deflections) were artificially generated using a standard layered elastic pavement model. A normally distributed random error with a standard deviation of 1.95μm (determined from multiple FWD drops) was added to each surface deflection and a back-analysis was performed using MODCOMP 2 to determine the layer stiffness moduli. The addition of this error resulted in COV's ranging from 0.2% for the deflection measurement directly under the load to 1.1% for the deflection measurement furthest from the load. These layer stiffness moduli were then compared to those used to generate the artificial deflection bowls. Irwin *et. al.* (1989) found that the mean value of the back-calculated stiffness modulus of the asphaltic material differed from the true value by approximately 2% and the variation in back-calculated stiffness modulus resulted in a Coefficient of Variation (COV) of approximately 16%. They also found that averaging deflection bowls prior to back-analysis significantly improved the accuracy of the resulting back-calculated stiffness moduli. Irwin *et. al.* (1989) also investigated the effects on back-calculated stiffness moduli of averaging deflection bowls obtained from repeat FWD drops prior to back-analysis. They found that averaging 3 and 5 deflection bowls reduced the COV to approximately 12% and 5% respectively. They concluded that three to five replicate drops at each height and each test point should be made to minimise the effect of random surface deflection errors.

Pronk (1988) also investigated the effect of surface deflection measurement inaccuracy on back-calculated stiffness moduli from a three layer flexible pavement system with a thicker asphalt layer (180mm) using a similar approach. Twenty deflection bowls (each containing 7 deflections) were generated using a standard layered elastic program and a random measurement error (within the supplier specified measuring error of 2% ± 2μm) was added to each deflection. (It should be noted that the 2% error is a systematic error which can be reduced by periodic calibration of the geophones.) The addition of this error resulted in deflection COV's ranging from 1.1% to 1.9%. A back-analysis was then performed and the resulting stiffness moduli were compared to those used to generate the deflection bowls. Results showed that the mean value of the back-calculated stiffness modulus of the asphaltic material differed from the true value by approximately 5% and the variation in back-calculated stiffness modulus resulted in a Coefficient of Variation (COV) of approximately 14%. Pronk (1988) also investigated the effects of averaging deflection bowls obtained from repeat FWD drops prior to back-analysis. He found that averaging 4 and 8 deflection bowls reduced the COV to approximately 8% and 5% respectively.

It should be noted that the effects of random surface deflection measurements on variability of the back-calculated stiffness modulus of the asphaltic material are not included in the analysis presented in the paper and should be considered separately.

CONCLUSIONS

- A model has been developed to statistically correct the stiffness modulus of the asphaltic material which is back-calculated from FWD measurements.
- The model accounts for variations in asphalt layer thickness and incorrect assumptions regarding the mean asphalt layer stiffness.
- The accuracy of the model has been assessed showing that the mean back-calculated FWD stiffness modulus will lie within –0.14% and +0.23% of the true mean stiffness modulus and the standard deviation of the back-calculated stiffness modulus will lie within –2.8% and +2.6% of the true mean stiffness modulus standard deviation for a confidence level of 95% when actual thicknesses are known.
- The model has been verified using a limited amount of published data.
- The model shows that the FWD stiffness modulus mean error is most sensitive to incorrect assumptions regarding the mean thickness of the asphalt layer $(h_0/\overline{h})$ and is relatively insensitive to asphalt layer thickness variability about a mean level $(\sigma_{\Delta h}/\overline{h})$.
- The model predicts that incorrect assumptions regarding the correct asphalt layer thickness $(h_0/\overline{h})$ in the range ±0.15 result in a FWD stiffness modulus mean error of between approximately –30% and +40%.
- The model shows that the FWD stiffness modulus standard deviation error is most sensitive to asphalt stiffness modulus variability $(\sigma_{\Delta E}/\overline{E})$ and asphalt layer thickness variability about a mean level $(\sigma_{\Delta h}/\overline{h})$ compared to assumptions regarding the mean thickness of the asphalt layer $(h_0/\overline{h})$.

- The model predicts that random asphalt thickness variations $(\sigma_{\Delta h}/\overline{h})$ in the range 0 to 0.15 result in a FWD stiffness modulus standard deviation error of between approximately –30% and +340%.
- For realistic distributions of asphalt stiffness modulus and asphalt layer thickness $(\sigma_{\Delta E}/\overline{E} = 0.1, \sigma_{\Delta h}/\overline{h} = 0.1)$ the model predicts that the error between the standard deviation of the asphaltic material stiffness modulus back-calculated from FWD results and the true asphaltic material stiffness modulus standard deviation is 150%.
- The ratio of the mean value of the back-calculated asphaltic material stiffness modulus calculated using 10 data points to characterise the asphalt layer thickness to the mean value of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness is between approximately 0.89 to 1.15 for a confidence level of 95% and between approximately 0.92 and 1.1 for a confidence level of 85% (assuming *m*= 2.3 and $\sigma_{\Delta h}/\overline{h} = 0.07$).
- The ratio of the standard deviation of the back-calculated asphaltic material stiffness modulus calculated using 10 data points to characterise the asphalt layer thickness to the standard deviation of the asphaltic material stiffness modulus calculated using an infinite number of data points to characterise the asphalt layer thickness is between approximately 0.7 and 1.89 for a confidence level of 95% and between approximately 0.76 and 1.57 for a confidence level of 85% (assuming *m* = 2.3, $\sigma_{\Delta h}/\overline{h} = 0.07$ and $\sigma_{\Delta E}/\overline{E} = 0.1$).

NOTATION

C_1, C_2, C_3	= positive constants;
d_1, d_2	= surface deflection directly under and 300mm from FWD load;
E[ ]	= expectation operator;
E	= asphaltic material stiffness modulus;
$\overline{E}$	= average stiffness modulus estimate;
E_{FWD}	= asphaltic material stiffness modulus from FWD;
$(\overline{E})^{true}$	= average stiffness modulus (infinite number of data points);
h	= asphalt layer thickness
$\overline{h}$	= constant asphalt layer thickness assumed in FWD back-analysis;
h_0	= difference between real mean asphalt layer thickness $(h_0 + \overline{h})$ and $\overline{h}$;
K, m	= positive constants;
N	= number of samples;
n	= N-1;
p	= positive constant;
S_b	= effective bending stiffness;
$t_{n;\alpha}$	= 100α percentage point of the student *t* distribution;
x	= longitudinal distance along pavement;
$\chi^2_{n;\alpha}$	= 100α percentage point of the χ^2 student *t* distribution;
ΔE	= zero mean asphaltic material stiffness modulus having variance $\sigma^2_{\Delta E}$;

Δh	= zero mean asphalt layer thickness having variance $\sigma^2_{\Delta h}$;
$\Phi^2_{E_{FWD}}$	= mean square of the back-analysed stiffness modulus;
$\sigma_{E_{FWD}}$	= standard deviation of asphaltic material stiffness modulus from FWD;
$(\sigma_{\Delta E})^{true}$	= standard deviation of stiffness modulus (infinite number of data points);
$\mu_{E_{FWD}}$	= mean FWD stiffness modulus;

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